Online Appendix
Technological Change and Obsolete Skills: Evidence from Men's Professional Tennis

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A Model

A.1 Player skills and quality

This section gives the details of the model of skill-altering technological change we use to guide our empirical analysis in the paper. The model is a simple overlapping generations model in the spirit of MacDonald and Weisbach (2004). Each cohort consists of a mass one of players, each of whom live for three periods. We index periods with t and a player's age with a. Let s_i denote the period player i is "born," so that a player's age in period t is just $a_i(t) = t - s_i + 1$. A player's performance at age a depends both on his skills and how these skills combine with the current racquet technology. Let $\{x_{ia}, y_{ia}\}$ denote player i's skills at age a (these skills might be thought of as being control and power/spin). A racquet technology r consists of the pair (A_r, λ_r), and a player's quality q_{iar} using racquet technology r is given by

$$q_{iar} = A_r x_{ia}^{\lambda_r} y_{ia}^{1-\lambda_r}$$

so that

$$\log q_{iar} = \log A_r + \lambda_r \log x_{ia} + (1 - \lambda_r) \log y_{ia}.$$

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Changes in the parameters A_r and λ_r affect players differently. The parameter $\lambda_r \in (0,1)$ controls the relative importance of skill x in using that racquet. A technology shock that decreases (increases) λ_r will help players who have relatively more (less) of skill y. Increases in A_r is an increase in total factor productivity and increases quality for every player. However, player performance is zero-sum—every win for one player corresponds to a loss for another player—and, under the assumptions we make in Section 6, an increase in A_r for all players will not affect the probability that one player defeats another.

A.2 Evolution of skill

Players choose an optimal mix of skill investment, and their skills evolve over time. Player i is born in period s_i with a common skill vector $\{x_0, y_0\} \in \mathscr{R}^2_+$.[1] At age a, he selects the optimal technology for his skill composition and improves his skills by $(\alpha_a + \varepsilon_{ia})\left(\lambda_r^2 + (1-\lambda_r)^2\right)^{-1/2}$ units in the $\log x$–$\log y$ plane, where the shock ε_{ia} is mean zero and iid with distribution F and $\left(\lambda_r^2 + (1-\lambda_r)^2\right)^{-1/2}$ serves as a normalizing constant.[2] A player chooses the optimal direction of improvement in $\log x$–$\log y$ space, which amounts to setting the direction of improvement perpendicular to the isoquant of the racquet technology he is using. Note that, although players choose the *direction* of investment in skills, the amount of improvement is determined exogenously. This modeling assumption focuses attention on the player's choice of direction of investment, rather than the level of investment. These assumptions give us the following laws of motion for skills:

$$\Delta \log x_{ia} = \log x_{ia} - \log x_{i,a-1} = \frac{\lambda_r}{\lambda_r^2 + (1-\lambda_r)^2}(\alpha_a + \varepsilon_{ia}) \tag{1}$$

$$\Delta \log y_{ia} = \log y_{ia} - \log y_{i,a-1} = \frac{1-\lambda_r}{\lambda_r^2 + (1-\lambda_r)^2}(\alpha_a + \varepsilon_{ia}) \tag{2}$$

[1] See Appendix Section C for a discussion about allowing for heterogeneous initial skills.

[2] It is worth noting that our model does not allow for skills to depreciate. Earlier versions of the paper had this feature, but it added complexity without adding any additional insights we could test in our data. An additional theoretical insight from allowing skills to depreciate is that the larger the rate of depreciation, the less the effect of a technological shock. Taken to its extreme, if skills depreciate completely each period then the technological shock has no effect on players.

These skill-specific laws of motion imply that, in the absence of any changes in racquet technology, the player's overall quality evolves according to

$$\begin{aligned}\Delta \log q_{iar} = \log q_{iar} - \log q_{i,a-1,r} &= \lambda_r \Delta \log x_{ia} + (1-\lambda_r)\Delta \log y_{ia} \\ &= \alpha_a + \varepsilon_{ia} \end{aligned} \tag{3}$$

We assume $\alpha_1 > \alpha_2 > \alpha_3 > 0$ so that a player's quality, in expectation, grows over his career and is concave. Heterogeneity in player quality arises over time because of the shocks ε_{ia}, since those who receive favorable shocks end up with higher quality than those who do not.

A.3 Player quality and earnings

At the beginning of each period, a player must choose whether to play tennis *before* observing his shock. If a player plays tennis in period t, he earns $q_{ia_i(t)r} \cdot p(Q_t)$, where $Q_t = \sum_j q_{ja_j(t)r}$, $p(Q)$ is continuous and differentiable, and $p'(Q) < 0$.[3] If he chooses not to play, then he earns a wage w_0.

A.4 Entry and exit

Each period unfolds in the following order:

1. Players decide whether to play or not. If a player does not play, he receives a wage w_0. Once he decides not to play, he may not rejoin the tour later.

2. Players decide which racquet technology to use.

3. Players choose their direction of investment. The optimal investment depends on λ_r, the relative weight the racquet places on one skill over another.

4. Players receive their quality shocks ε_{ia} and their new qualities are realized $\log q_{iar} = \log q_{i,a-1,r} + \alpha_a + \varepsilon_{ia}$.

5. The player, if he plays tennis, receives his wage $q_{iar} \cdot p(Q_t)$.

Players choose whether to play, which technology to use, and the direction of their investment to maximize their expected earnings. Recall that players only live for three periods.[4]

[3] The summation in the expression for Q_t is over all players who are playing tennis in period t.

[4] Allowing for an arbitrary number of periods does not change our results.

The present value of a player's earnings when he is age a at the start of period t is

$$V_a^t\left(q_{a-1}, \vec{z}_{t-1}\right) = \max\left\{ \underbrace{\mathbb{E}\left[q_a \cdot p(Q_t) + \beta V_{a+1}^{t+1}(q_a, \vec{z}_t) \mid q_{a-1}, \vec{z}_{t-1}\right]}_{\text{expected value of playing for one more year}}, \underbrace{\sum_{\tau=a}^{3} \beta^{\tau-a} w_0}_{\text{outside option}} \right\}, \tag{4}$$

for $a \in \{1,2,3\}$. In the final year of a player's career the continuation value is zero:

$$V_4^{t+1}\left(q_3, \vec{z}_t\right) = 0.$$

The vector of aggregate state variables is $\vec{z}_{t-1} = (m_{1,t-1}, m_{2,t-1}, \widehat{q}_{2,t-1})$. m_{1t} and m_{2t} denote the masses of young and middle-aged players who choose to play in period t. $\widehat{q}_{2t}$ denotes the threshold quality for participation of middle-aged players, so that a middle-aged player in period t plays if, and only if, q_{i1} is above $\widehat{q}_{2t}$. Last period's quality threshold for middle-aged players appears as a state variable because it affects the quality of old players in period t. A similar threshold exists for old players, but it does not enter as a state variable because the players who were old last period are no longer playing tennis. An individual player's quality appears as a state variable for middle-age and old players. It does not appear for young players because young players all start the period with the same quality q_0.

The participation thresholds $\widehat{q}_{2t}$ and $\widehat{q}_{3t}$ are defined by the requirement that a player at the threshold is indifferent between playing or not:

$$[\widehat{q}_{2t}]: \qquad \mathbb{E}\left[q_2 \cdot p(Q_t) + \beta V_3^{t+1}\left(q_2, \vec{z}_t\right) \mid q_1 = \widehat{q}_{2t}, \vec{z}_{t-1}\right] = (1+\beta) w_0, \tag{5}$$

$$[\widehat{q}_{3t}]: \qquad \mathbb{E}\left[q_3 \cdot p(Q_t) \mid q_2 = \widehat{q}_{3t}, \vec{z}_{t-1}\right] = w_0. \tag{6}$$

Since young players are identical when they decide whether to play or not, the mass of players who enters, m_{1t}, is determined by the requirement that all young players are indifferent between playing or not:

$$[m_{1t}]: \qquad \mathbb{E}\left[q_1 \cdot p(Q_t) + \beta V_2^{t+1}\left(q_1, \vec{z}_t\right) \mid \vec{z}_{t-1}\right] = (1+\beta+\beta^2) w_0. \tag{7}$$

We assume that there is always an interior solution for m_{1t}.

The masses of middle-aged and old players that choose to play tennis in period t, are given by

$$m_{2t} = m_{1,t-1} \cdot \mathbb{P}\left[q_1 \geq \widehat{q}_{2t}\right], \text{ and}$$
$$m_{3t} = m_{1,t-2} \cdot \mathbb{P}\left[q_1 \geq \widehat{q}_{2,t-1} \wedge q_2 \geq \widehat{q}_{3t}\right].$$

Putting these expressions together, the total quality of all players, Q_t, is given by

$$Q_t = m_{1t}\,\mathbb{E}\left[q_1\right] + m_{2t}\,\mathbb{E}\left[q_2 \mid q_1 \geq \widehat{q}_{2t}\right] + m_{3t}\,\mathbb{E}\left[q_3 \mid q_1 \geq \widehat{q}_{2,t-1} \wedge q_2 \geq \widehat{q}_{3t}\right].$$

A.5 Three useful lemmas

We now present three lemmas that will be useful later. All three are fairly intuitive, and their proofs are found in section B of the appendix.

Lemma 1. *V_a^t is increasing in player quality for all $a \in \{1,2,3\}$. That is,*

$$q' > q \Rightarrow V_a^t\left(q', \vec{z}_{t-1}\right) \geq V_a^t\left(q, \vec{z}_{t-1}\right).$$

Moreover, these inequalities are strict if the player chooses to play in period t rather than take the outside option.

Lemma 2. *There exists a unique value for $\widehat{q}_{2t}$ (and $\widehat{q}_{3t}$) such that middle-aged (old) players in period t choose to play when $q > \widehat{q}_{2t}$ ($q > \widehat{q}_{3t}$), while those with quality $q < \widehat{q}_{2t}$ ($q < \widehat{q}_{3t}$) choose to exit and take the outside option.*

Lemma 3. *Suppose the market is in a steady state. Then $\widehat{q}_{3t} \geq \widehat{q}_{2t} \geq \widehat{q}_{1t}$. That is, as players age their exit cutoffs also (weakly) rise.*

The intuition for Lemma 3 is that younger players are willing to play with a lower quality in the current period, even if expected earnings are below w_0, because they anticipate enjoying a higher quality, and thus higher earnings, in the future. In contrast, old players have no future period so they will only be willing to play if their expected earnings are above w_0.

B Proofs

B.1 Proof of Lemma 1

Proof. When a player switches to the new racquet his quality grows according to

$$\begin{aligned}
\Delta \log q'_{ia} &= \log q'_{ia} - \log q_{i,a-1} \\
&= \log A' - \log A + \lambda' \log x_{ia} - \lambda \log x_{i,a-1} \\
&\quad + (1-\lambda') \log y_{ia} - (1-\lambda) \log y_{i,a-1} \\
&= \alpha_a + \varepsilon_{ia} + \log A' - \log A + \left(\lambda' - \lambda\right)(\log x_{i,a-1} - \log y_{i,a-1}) \qquad \text{(B.1)}
\end{aligned}$$

where q'_{ia} is player i's quality using the new racquet. Defining

$$u_{ia} = \log A' - \log A + \left(\lambda' - \lambda\right)(\log x_{i,a-1} - \log y_{i,a-1}), \qquad \text{(B.2)}$$

(B.1) becomes

$$\Delta \log q'_{ia} = \alpha_a + \varepsilon_{ia} + u_{ia} = \Delta \log q_{ia} + u_{ia} \qquad \text{(B.3)}$$

■

B.2 Proof of Lemma 1

Proof. We prove this by induction, and so begin with V_3^t. First, note that $\mathbb{E}[q_3 \cdot p(Q_t) \mid q_2, \vec{z}_{t-1}]$ is strictly increasing in q_2, while the wage of the outside option w_0 is unrelated to q_2. Thus, V_3^t is weakly increasing in q_2 and strictly increasing if the player chooses to play in period t.

Now we turn to V_a^t and assume V_{a+1}^{t+1} is increasing in player quality. First, note that $\mathbb{E}\left[q_a \cdot p(Q_t) + \beta V_{a+1}^{t+1}(q_a, \vec{z}_t) \mid q_{a-1}, \vec{z}_{t-1}\right]$ is weakly increasing in q_{a-1}. This is true because $\mathbb{E}[q_t \cdot p(Q_t) \mid q_{a-1}, \vec{z}_{t-1}]$ is strictly increasing in q_{a-1} while $\mathbb{E}\left[\beta V_{a+1}^{t+1}(q_a, \vec{z}_t) \mid q_{a-1}, \vec{z}_{t-1}\right]$ is weakly increasing in q_{a-1}. As before, the present value of the outside option, $\sum_{\tau=0}^{3-a} \beta^\tau w_0$, is unrelated to q_{a-1}. Thus, V_a^t is weakly increasing in q_a and strictly increasing if the player chooses to play in period t. ■

B.3 Proof of Lemma 2

Proof. An old player ($a = 3$) entering period t with quality q_2 will continue playing if, and only if, the value of playing is no less than the outside option:

$$\mathbb{E}\left[q_3 \cdot p(Q_t) \mid q_2, \vec{z}_{t-1}\right] \geq w_0. \tag{B.4}$$

Note that

$$\lim_{q_2 \to 0} \mathbb{E}\left[q_3 \cdot p(Q_t) \mid q_2, \vec{z}_{t-1}\right] = 0$$
$$\lim_{q_2 \to \infty} \mathbb{E}\left[q_3 \cdot p(Q_t) \mid q_2, \vec{z}_{t-1}\right] = \infty.$$

Also, note that the left-hand side of (B.4) is a strictly increasing, continuous function of q_2. Thus, as long as w_0 is positive and finite, equation (B.4) will hold with equality at exactly one point. Therefore, $\widehat{q}_{3t}$ exists and is unique.

A middle-aged player entering period t with quality q_1 will continue playing if, and only if, the value of playing for one more period is no less than the present value of the outside option:

$$\mathbb{E}\left[q_2 \cdot p(Q_t) + \beta V_3^{t+1}\left(q_2, \vec{z}_t\right) \mid q_1, \vec{z}_{t-1}\right] \geq (1+\beta) w_0. \tag{B.5}$$

The first term of the left-hand side of (B.5) is strictly increasing in q_1, while from Lemma 1 we know that the second term is weakly increasing in q_1. Thus, the entire term is strictly increasing in q_1. It also likewise follows that it is continuous. Following the same reasoning as the previous paragraph, it must be that $\widehat{q}_{2t}$ exists and is unique. ■

B.4 Proof of Lemma 3

Proof. We first show $\widehat{q}_{3t} \geq \widehat{q}_{2t}$. Since we are in a steady state, we drop the dependence on t. By way of contradiction, suppose that $\widehat{q}_2 > \widehat{q}_3$. Consider a quality level $q^* \in (\widehat{q}_3, \widehat{q}_2)$. Then it must be the case that

$$\mathbb{E}\left[q_2 \cdot p(Q) + \beta V_3\left(q_2, \vec{z}\right) \mid q_1 = q^*, \vec{z}\right] < (1+\beta) w_0 \tag{B.6}$$
$$\mathbb{E}\left[q_3 \cdot p(Q) \mid q_2 = q^*, \vec{z}\right] > w_0 \tag{B.7}$$

From (B.6) we can write

$$
\begin{aligned}
(1+\beta)w_0 &> \mathbb{E}\left[q_2 \cdot p(Q) \mid q_1 = q^*, \vec{z}\right] + \beta\,\mathbb{E}\left[V_3\big(q_2, \vec{z}\big) \mid q_1 = q^*, \vec{z}\right] \\
&\geq \mathbb{E}\left[q_2 \cdot p(Q) \mid q_1 = q^*, \vec{z}\right] + \beta w_0
\end{aligned}
$$

Combining this with (B.7) we get

$$
\mathbb{E}\left[q_2 \cdot p(Q) \mid q_1 = q^*, \vec{z}\right] < w_0 < \mathbb{E}\left[q_3 \cdot p(Q) \mid q_2 = q^*, \vec{z}\right]
$$

But this is a contradiction because the distribution of $q_2 \mid q_1 = q^*$ first-order stochastically dominates the distribution of $q_3 \mid q_2 = q^*$, which means it must be the case that

$$
\mathbb{E}\left[q_2 \cdot p(Q) \mid q_1 = q^*, \vec{z}\right] > \mathbb{E}\left[q_3 \cdot p(Q) \mid q_2 = q^*, \vec{z}\right].
$$

which is a contradiction. It is straightforward to extend this proof to show $\widehat{q}_{2t} \geq \widehat{q}_{1t}$. ■

B.5 Proof of Proposition 1

Proof. As players improve, they invest more in the skill that the racquet they are using puts more weight on. From (1) and (2) we know

$$
\log x_{ia} - \log y_{ia} = \log x_{i,a-1} - \log y_{i,a-1} + \frac{2\lambda - 1}{\lambda^2 + (1-\lambda)^2}\left(\alpha_a + \varepsilon_{ia}\right). \tag{B.8}
$$

By iterating equation (B.8), we find

$$
\log x_{ia} - \log y_{ia} = \log x_0 - \log y_0 + \frac{2\lambda - 1}{\lambda^2 + (1-\lambda)^2} \sum_{\tau=1}^{a} \left(\alpha_\tau + \varepsilon_{i\tau}\right). \tag{B.9}
$$

Similarly, iterating equation (3) gives

$$
\begin{aligned}
\log q_{ia} &= \log q_0 + \sum_{\tau=1}^{a} \left(\alpha_\tau + \varepsilon_{i\tau}\right) \\
\Rightarrow \sum_{\tau=1}^{a} \left(\alpha_\tau + \varepsilon_{i\tau}\right) &= \log q_{ia} - \log q_0.
\end{aligned} \tag{B.10}
$$

Substituting (B.10) into (B.9) yields

$$\log x_{ia} - \log y_{ia} = \log x_0 - \log y_0 + \frac{2\lambda - 1}{\lambda^2 + (1-\lambda)^2} \left(\log q_{ia} - \log q_0 \right). \tag{B.11}$$

Substituting (B.11) into (B.2) gives us

$$u_{ia} = \log A' - \log A + \left(\lambda' - \lambda\right) \left(\log x_0 - \log y_0\right) + \left(\lambda' - \lambda\right) \frac{2\lambda - 1}{\lambda^2 + (1-\lambda)^2} \left(\log q_{i,a-1} - \log q_0\right). \tag{B.12}$$

This implies

$$u_{ia_i(t)} - u_{ja_j(t)} = \left(\lambda' - \lambda\right) \frac{2\lambda - 1}{\lambda^2 + (1-\lambda)^2} \left(\log q_{i,a_i(t-1)} - \log q_{j,a_j(t-1)}\right). \tag{B.13}$$

Since $\lambda' < \lambda$ and $\lambda > 0.5$, (B.13) implies that $u_{ia_i(t)} < u_{ja_j(t)}$. ■

B.6 Proof of Corollary 1

Proof. First note that equation (3) implies

$$\log q_{ia_i(t)} - \log q_{ja_j(t)} = \log q_{i,a_i(t-1)} - \log q_{j,a_j(t-1)} + \alpha_{a_i(t)} - \alpha_{a_j(t)} + \varepsilon_{ia_i(t)} - \varepsilon_{ja_j(t)} \tag{B.14}$$

and equations (B.3) and (B.12) imply

$$\begin{aligned} \log q'_{ia_i(t)} - \log q'_{ja_j(t)} &= \log q_{ia_i(t)} - \log q_{ja_j(t)} + u_{ia_i(t)} - u_{ja_j(t)} \\ &= \underbrace{\frac{\lambda\lambda' + (1-\lambda)(1-\lambda')}{\lambda^2 + (1-\lambda)^2}}_{B} \left(\log q_{i,a_i(t-1)} - \log q_{j,a_j(t-1)}\right) \\ &\quad + \alpha_{a_i(t)} - \alpha_{a_j(t)} + \varepsilon_{ia_i(t)} - \varepsilon_{ja_j(t)}. \end{aligned} \tag{B.15}$$

Since $\lambda > .5$ and $\lambda' < \lambda$, it must be the case that $0 \leq B < 1$. Equation (B.14) implies

$$\begin{aligned} &\mathbb{P}\left[\log q_{ia_i(t)} > \log q_{ja_j(t)}\right] \\ &\quad = \mathbb{P}\left[\varepsilon_{ia_i(t)} - \varepsilon_{ja_j(t)} > -\left(\log q_{i,a_i(t-1)} - \log q_{j,a_j(t-1)} + \alpha_{a_i(t)} - \alpha_{a_j(t)}\right)\right], \end{aligned} \tag{B.16}$$

while equation (B.15) implies

$$\begin{aligned}
&\mathbb{P}\left[\log q'_{ia_i(t)} > \log q'_{ja_j(t)}\right] \\
&\quad = \mathbb{P}\left[\varepsilon_{ia_i(t)} - \varepsilon_{ja_j(t)} > -\left(B\left(\log q_{i,a_i(t-1)} - \log q_{j,a_j(t-1)}\right) + \alpha_{a_i(t)} - \alpha_{a_j(t)}\right)\right]. \quad \text{(B.17)}
\end{aligned}$$

Equations (B.16), (B.17), and $B < 1$ together imply

$$\begin{aligned}
&\mathbb{P}\left(q'_{ia_i(t)} > q'_{ja_j(t)} \mid q_{i,a_i(t-1)} > q_{j,a_j(t-1)}\right) \\
&\qquad\qquad < \mathbb{P}\left(q_{ia_i(t)} > q_{ja_j(t)} \mid q_{i,a_i(t-1)} > q_{j,a_j(t-1)}\right). \quad \text{(B.18)}
\end{aligned}$$

Since Kendall's τ has the property that $E(\tau) = \mathbb{P}(x_{it} > x_{jt} \mid x_{i,t-1} > x_{j,t-1}) - \mathbb{P}(x_{it} < x_{jt} \mid x_{i,t-1} > x_{j,t-1})$, equation (B.18) implies that the introduction of the new racquet reduces the rank correlation of period-to-period player quality in the period when the racquet is introduced. ■

B.7 Proof of Proposition 2

Proof. Given that we are in a steady state, we know from Lemma 3 that the distribution of quality for older players has a higher truncation point than the distribution for younger players. Moreover, the quality of older players is shifted right, relative to that of younger players, by α_2 and/or α_3. For both reasons, the median quality of older players in period t is greater than the median quality of younger players in period t. This implies that $\mathbb{P}\left[q_{ia_i(t)} > q_{ja_j(t)} \mid a_i(t) > a_j(t)\right] > .5$. By Proposition 1, this implies $\mathbb{P}\left[u_{ia_i(t)} < u_{ja_j(t)} \mid a_i(t) > a_j(t)\right] > .5$. ■

B.8 Proof of Proposition 3

Proof. We show the effect of the technological shock on the exit rates of the oldest players is ambiguous by considering the two examples described in the text. First, we show it is possible for the exit rate for the oldest players to decrease. If the demand for tennis output is perfectly elastic, $p'(Q) = 0$, then the price per unit of quality is unchanged by the new technology, and so, by (6), $\widehat{q}'_{3t} = \widehat{q}_{3t}$. By assumption, the new racquet is enough better that all players switch in the same period, so that $q'_{i,2} > q_{i,2} \forall i$. Thus there exists an $\varepsilon > 0$ such that all players i for whom $q_{i,2} \in (\widehat{q}_{3t} - \varepsilon, \widehat{q}_{3t})$, and so were going to exit, now choose to continue playing since $q'_{i,2} \geq \widehat{q}'_{3t}$. Thus, it is possible for the exit rate for the oldest players to decrease.

Next, we show it is possible for the exit rate for the oldest players to increase. Consider another extreme case where demand is not perfectly elastic, but $\lambda = 1$ and $\lambda' = 0$. The technological shock completely devalues all investments any of the oldest players have made and their quality is now equal to that of the youngest players: $q_{i,2} = q_0$.

However, by (4), $V_2^{t+1}(q_1, \vec{z}_t) \geq (1+\beta)w_0$ for all possible q_1 and $\vec{z}_t$, with the inequality strict for some q_1 and $\vec{z}_t$. Thus, as long as the probability that a new player continues to play in the second period is greater than zero, then

$$\mathbb{E}\left[V_2^{t+1}(q_1, \vec{z}_t) \mid q_0, \vec{z}_{t-1}\right] > (1+\beta)w_0.$$

Therefore, (7) implies a new player's expected earnings are less than w_0:

$$\mathbb{E}\left[q_1 \cdot p(Q_t) \mid q_0, \vec{z}_{t-1}\right] < w_0. \tag{B.19}$$

By (3), if an old and young player start the period at the same quality, the young player's expected quality at the end of the period is greater than the old player's expected quality at the end of the period:

$$\mathbb{E}\left[q_1 \mid q_0 = q_0, \vec{z}_{t-1}\right] > \mathbb{E}\left[q_3 \mid q_2 = q_0, \vec{z}_{t-1}\right].$$

Therefore, the old player's expected earnings in the current period are less than the young player's expected earnings, which is in turn less than w_0 by (B.19):

$$w_0 > \mathbb{E}\left[q_1 \cdot p(Q_t) \mid q_0, \vec{z}_{t-1}\right] > \mathbb{E}\left[q_3 \cdot p(Q_t) \mid q_2 = q_0, \vec{z}_{t-1}\right].$$

Therefore, by (6), $\widehat{q}_{3t} > q_0$ and all old players exit. ■

C Discussion of heterogeneous initial skills

In general, allowing for heterogeneous initial skills complicates the analysis without adding additional insight. However, an alternative model we could consider is one where players are born with heterogeneous skills, and, while players could improve both skills, they could not change their initial mix of skills. In this model a change in racquet technology would change the type of players who enter. This alternative model leads to an important additional interpretation of our results: that technological change can hurt some workers who are born without the skills needed to succeed.

This alternative model has many of the same predictions as our model, however it leads to a very different prediction regarding player entry and exit. The alternative model predicts that young players who entered before the racquet change will exit at higher rates when the new racquet is introduced. The intuition for this is that while older players see their quality fall, given their lifetime of investment, many of them can still profitably play while the new players, whose endowment of skill better match the racquet, develop their skills. On the other hand, many young players who entered before the racquet change were playing at a loss, relative to their outside option, to develop the skills to earn profits later in their careers. With the racquet change, they no longer have the endowment of skills needed to have a viable path toward earning profits, and so will exit.

Put differently, in the alternative model, the new racquet leads to a much larger decrease in the continuation value for existing players. And this decrease in continuation values is more pronounced for younger players. At its extreme, this would lead to nearly all players in the second year of their professional career exiting.

As Figure 15 shows, we find that exit rates for the young are basically unchanged, while exit rates for the old climb significantly. This suggests that the alternative model does not match the data from professional tennis.

D Additional tables

Table D.1: Player-year summary statistics (not in SCC)

	Mean	Median	Std. dev.
Age	22.5	21.6	5.10
Annual singles wins	0.23	0	0.56
Annual singles matches	1.80	1	1.43
Observations	16313		

Notes: These summary statistics are only calculated for player-years not in the strongly connected component, but include all of their wins and matches, regardless of whether their opponent was in the strongly connected component.

Table D.2: Number of player-years excluded from the SCC

	Annual losses				
Annual wins	0	1	2	3+	Total
0	0	9,729	2,279	1,362	13,370
1	186	1,043	482	622	2,333
2	61	125	103	180	469
3+	28	37	14	62	141
Total	275	10,934	2,878	2,226	16,313

Notes: This table calculates the number of player-years in each cell that are excluded from the strongly connected component.

Table D.3: Elasticity of cohort prize money with respect to cohort R16 Grand Slam appearances

	(1)	(2)
log(Appearances)	1.212***	1.214***
	(0.0428)	(0.0370)
Observations	68	68
R^2	0.924	0.969
Year FE	No	Yes

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: The table reports results from regressing a birth cohort's (log) prize money in year t on its R16 Grand Slam appearances in year t. The sample is restricted to five-year birth cohorts between 1970–1989 and years 1990–2015.

Figure D.1: Total prize money over time

Notes: The figure plots the total prize money at Wimbledon and Roland-Garros between 1970–1993. Prize money was converted to U.S. dollars and corrected for inflation. The dashed lines mark the years when professional tennis players were transitioning to composite racquets.

E Results limited to Grand Slams

In this section we limit our data to the four Grand Slam tournaments. Because the Australian Open has changed the size of its initial round over time, we further limit ourselves to the round of 64 and later rounds. This has the benefit of giving us a consistent set of tournaments over our entire sample and allows us to address the possibility that our results are being driven by changes in the number of tournaments. Limiting ourselves to the Grand Slams also has a cost, as it reduces our sample and makes it impractical to estimate player quality.[5] Instead, we proxy for player quality using the number of Grand Slam matches in which a player competes.[6] We find that we can reproduce our results on this more limited data set. In some cases, the magnitude of the changes are smaller than in our main analysis (Figures E.5, E.6, E.8), while in others the changes are larger (Figures E.2 and E.3).

Several figures in our main analysis already limited the sample to Grand Slams, and we do not reproduce these results. This includes Figures 8, 9, and 19. We also do not reproduce results involving prize money, including Figures 13 and 14.

[5] We can only estimate player quality for between 15 and 50 percent of the players in any given year.

[6] An alternative measure of player quality is the number of matches they win. Given the tournament structure of tennis, this is highly correlated with the number of matches they compete in, with a correlation coefficient of 0.97.

E.1 Evidence for skill-altering technical change

Figure E.2 is very similar to Figure 6, and shows that the returns to height increased when the composite racquets were introduced. This is evidence that the composite racquets were a skill-altering technical change.

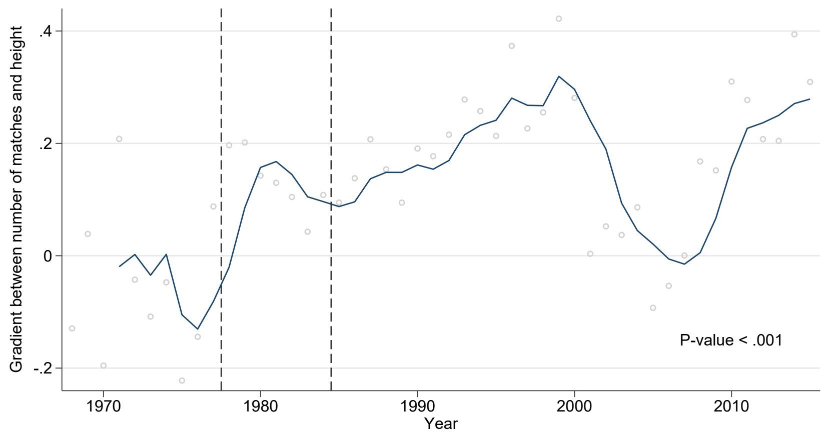

Figure E.2: Relationship between number of matches played and height over time

Notes: The figure plots the regression coefficient from regressing a player's number of matches played in Grand Slam tournaments on his height (in inches) in each year (circles) and a four-year lagged moving average (solid line). The dashed lines mark the years when professional tennis players were transitioning to composite racquets. The figure reports the p-value of a t-test of whether the return to height before 1978 differed from the return between 1985–1999.

E.2 Year-to-year rank correlation of player quality fell temporarily

Again, Figure E.3 is largely the same as Figure 7, and shows a drop in the year-to-year rank correlation of player quality, as proxied by the player's number of Grand Slam matches.

E.3 Younger players gained relative to older players

Figure E.4 is very similar to Figure 10, and shows that the transition to the composite racquets increased the share of matches played by young players and decreased the share of matches played by older players.

In Figure E.5 we plot the median age of players separately for those who are above or below the median number of Grand Slam matches for that year. While Figure E.5 shows

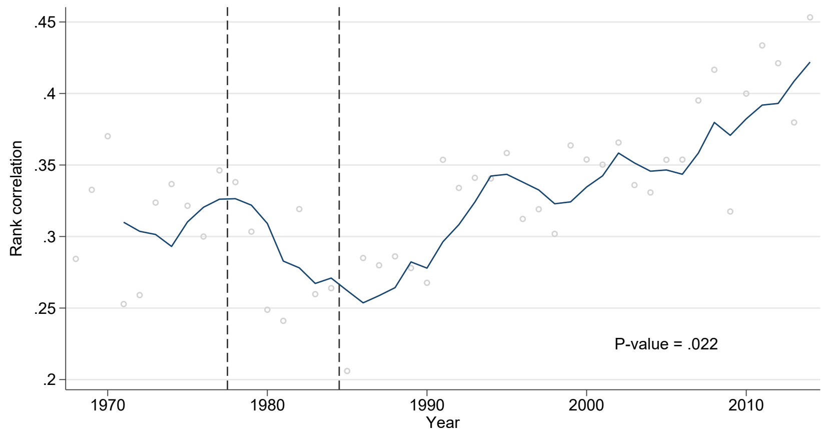

Figure E.3: Year-to-year rank correlation of number of matches played

Notes: This figure plots a measure of rank correlation (Kendall's τ) between a player's number of Grand Slam matches in consecutive years (circles). If a player did not appear in year t but did in year $t-1$ or $t+1$, we imputed zero matches in year t. The solid line plots a four-year lagged moving average. The vertical dashed lines mark the years of the racquet transition (1978–1984). The reported p-value corresponds to an F-test for whether the pre-1978, 1978–1984, and 1985–1999 periods have the same slopes and intercepts.

a similar pattern as Figure 11, the age gap between the median above- and below average player is significantly smaller we cannot reject the null hypothesis that the trendlines are parallel between 1970–1999. This may be due to the fact that, by limiting ourselves to Grand Slams, we are already restricted to the best players on tour. Thus, we don't find a stark difference between the best players and the *very* best players.

Figure E.6 shows, as does Figure 12, that the benefit of age is at its lowest in the mid 1980s. However, the difference between the pre-transition and post-transition gradient is less pronounced than in Figure 12.

E.4 Exit rates of older players rose relative to younger players

Figure E.7 shows the same patterns as Figure 15, showing that the exit rates for older players increased during the transition. Note that an "exit" in Figure E.7 means failing to qualify for a Grand Slam in year $t+1$ whereas in Figure 15, in the main text, an "exit" meant failing to be included in the strongly connected component in year $t+1$.

While Figure E.8 has a local minimum between 10 and 15 years, the right side of the figure does not show the dramatic rise that Figure 16 does and we cannot reject the hypothesis of a single linear line fitting the data. These results are a little quirky and difficult

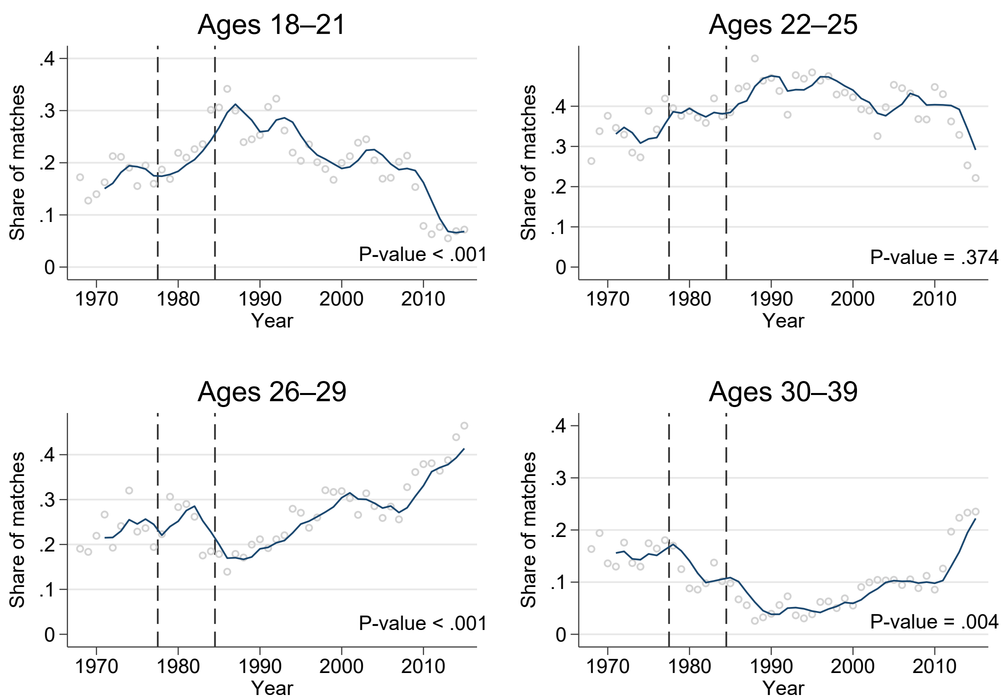

Figure E.4: Share of Grand Slam matches by age group over time

Notes: The figure plots the share of Grand Slam matches by age group for each year (circles) and a four-year lagged moving average (solid line). The vertical dashed lines mark the years of the racquet transition (1978–1984). The reported p-value corresponds to an F-test for whether the pre-1978, 1978–1984, and 1985–1999 periods have the same slopes and intercepts.

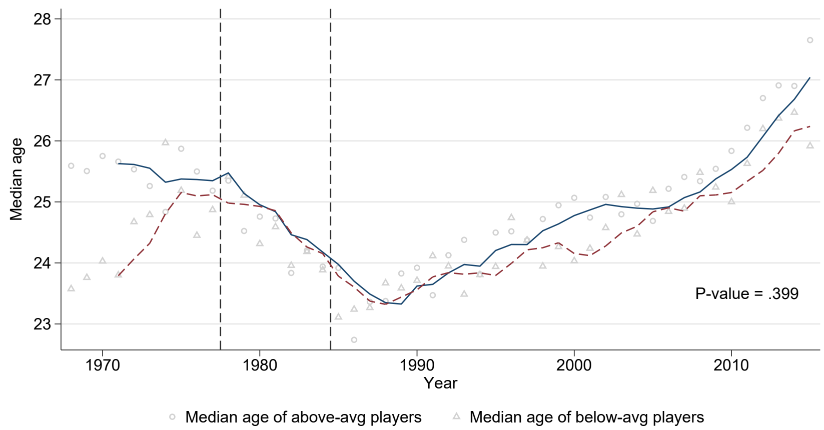

Figure E.5: Median age of above- and below-average players over time

Notes: The figure plots the median age of above- and below-average players in each year (circles and triangles) and a four-year lagged moving average (solid and dashed lines). Above- and below-average are defined using the median number of Grand Slam matches played in the given year. The vertical dashed lines mark the years of the racquet transition (1978–1984). The reported p-value corresponds to an F-test for whether above-average and below-average players have parallel trends between 1970–1999.

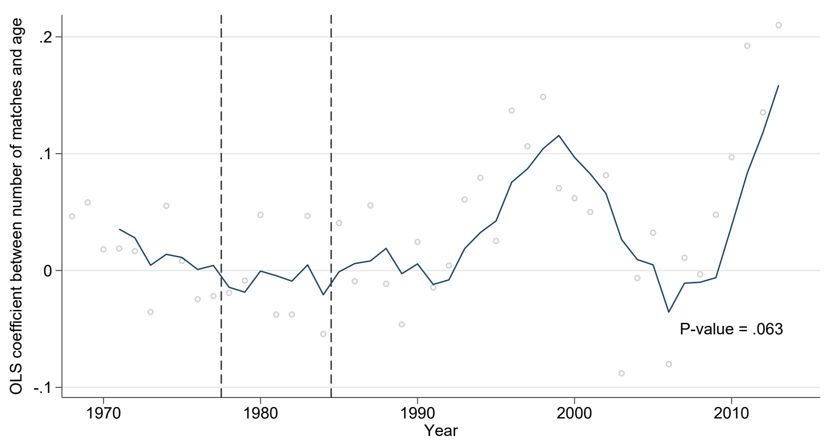

Figure E.6: Relationship between number of Grand Slam matches and age over time

Notes: The figure plots the age-matches gradient in each year (circles) and a four-year lagged moving average (solid line). The vertical dashed lines mark the years of the racquet transition (1978–1984). The reported p-value corresponds to an F-test for whether the pre-1978, 1978–1984, and 1985–1999 periods have the same slopes and intercepts.

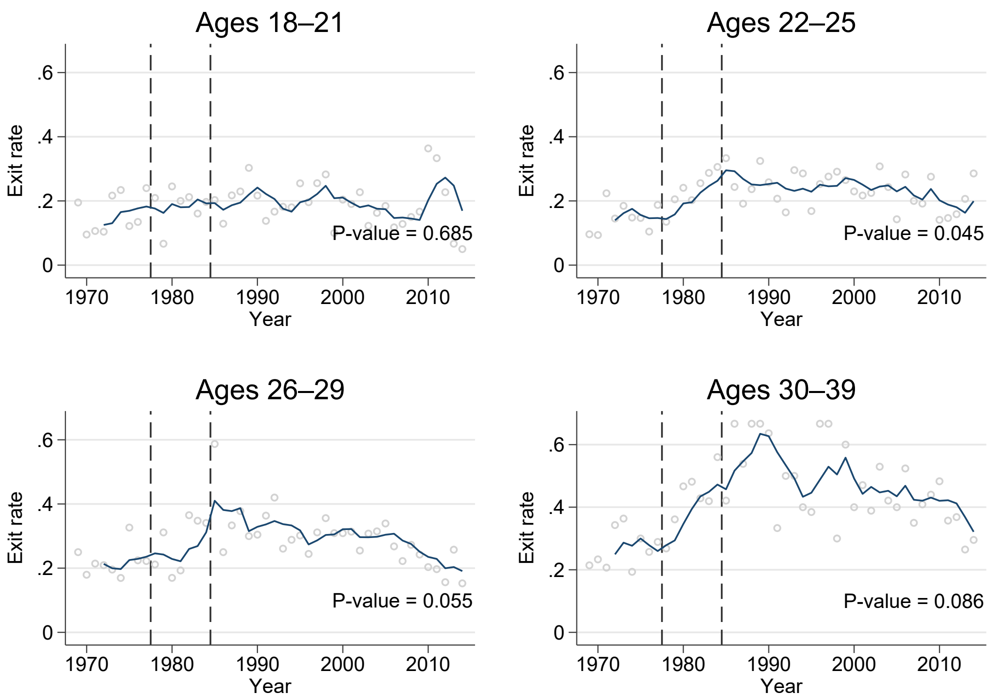

Figure E.7: Exit rates by age group over time

Notes: The figure plots the exit rate by age group for each year (circles) and a four-year lagged moving average (solid line). A player "exits" in year t if he plays at least one Grand Slam match in year t but does not play any Grand Slam matches in year $t+1$. The vertical dashed lines mark the years of the racquet transition (1978–1984). The reported p-value corresponds to an F-test for whether the pre-1978, 1978–1984, and 1985–1999 periods have the same slopes and intercepts.

to interpret. While the average age a player first competed in a Grand Slam is typically a year later than the average age a player first entered the tour for those who were 10 or younger in 1980, for those who were 21 or older in 1980, the average age a player first competed in a Grand Slam is *lower* than the average age a player first was on the tour. The seeming contradiction, that players are competing in Grand Slams before they are on the tour, comes from the sample selection rules. The data in the main text uses all players in the SCC while this section limits the data to those players who ever competed in a round of 64 in a Grand Slam.

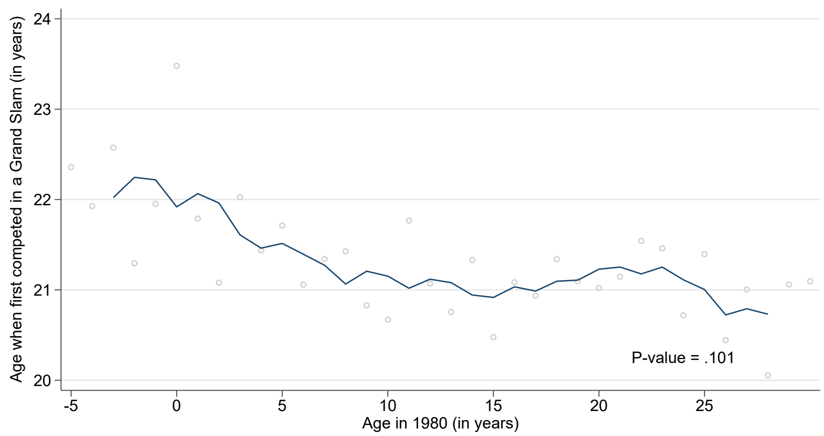

Figure E.8: Age when first competed in Grand Slam by age in 1980

Notes: The figure plots the mean age when a player first competed in a Grand Slam for each birth year, reported as age in 1980 (circles) and a five-year centered moving average (solid line). The vertical dashed lines mark the years of the racquet transition (1978–1984). The reported p-value corresponds to an F-test for whether the trend for points to the left of 10 have a different intercept and slope than the trend for points to the right of 10.

Figure E.9 shows, as does Figure 17, that the age distribution shifted younger during the transition to composite racquets.

E.5 Cross-sectional inequality during the transition

Figure E.10 is comparable to Figure 18, and shows a small dip in cross-sectional inequality during the transition.

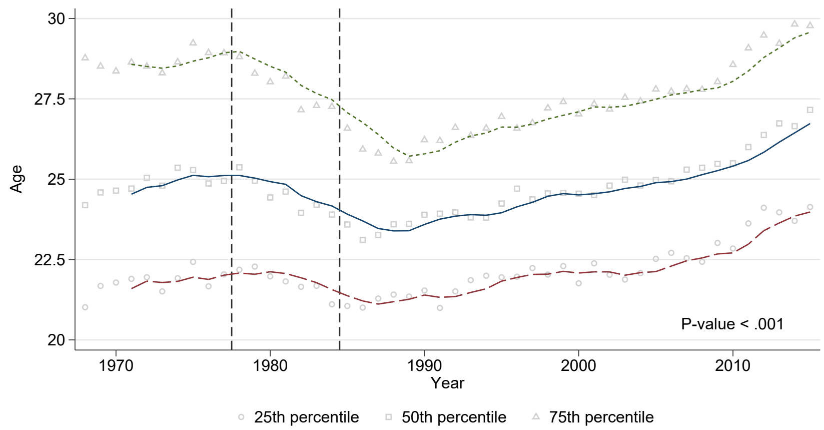

Figure E.9: Age distribution over time

Notes: The figure plots the quartiles of the age distribution of players competing in the Grand Slams in each year, along with four-year lagged moving averages. The vertical dashed lines mark the years of the racquet transition (1978–1984). The reported p-value corresponds to an F-test for whether the pre-1978, 1978–1984, and 1985–1999 periods have the same slopes and intercepts for the median age.

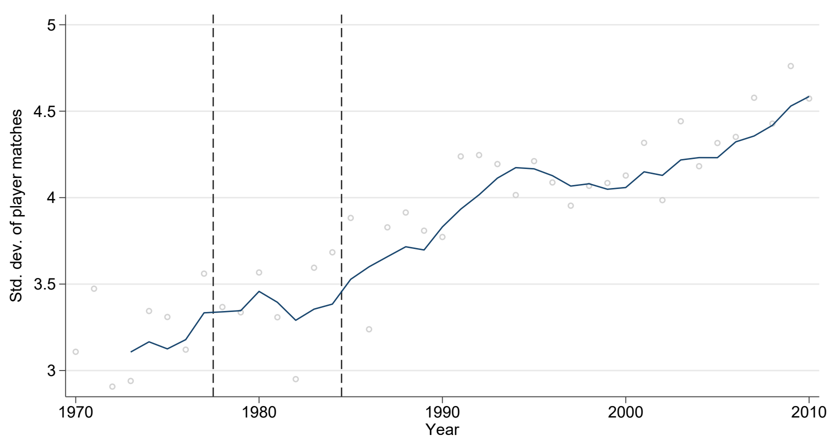

Figure E.10: Cross-sectional standard deviation of player Grand Slam matches

Notes: The figure plots the standard deviation of players' Grand Slam matches in each year, along with four-year lagged moving averages. The vertical dashed lines mark the years of the racquet transition (1978–1984)

References

MacDonald, G. and M. S. Weisbach (2004). The Economics of Has-beens. *Journal of Political Economy 112*(S1), S289–S310.